

Indian Statistical Institute, Bangalore

B. Math. Second Year

Second Semester - Graph Theory

Mid-Semester Exam

Duration: 3 hours

Date : March 03, 2015

Max Marks: 100

1. (a) If G is a graph with n vertices and $e \geq 1$ edges, and $d_1 \geq \dots \geq d_n$ are the degrees of its vertices, then what are the possible values of the ratio $\frac{d_1 + \dots + d_n}{e}$?
(b) Show that any tree with at least two vertices has at least two leaves. [10+10 = 20]
2. What are the values of n for which the complete graph K_n is planar ? [20]
3. (a) Define the graphical metric d_G of a connected graph G . Prove that it is really a metric.
(b) Let $2 \leq k \leq \frac{v}{2}$. Consider the graph $G = J(v, k)$ whose vertices are the k - subsets of a v - set; two vertices A, B are adjacent if $\#(A \cap B) = k - 1$. Show that $d_G(A, B) = \frac{1}{2}\#(A \Delta B)$ for any two vertices A, B of G . [8+12 = 20]
4. A perfect matching in a graph is a partition of its vertex set into edges. A factorisation is a partition of its edge set into perfect matchings.
(a) Find the total number of perfect matchings of the Peterson graph. Does it have a factorisation?
(b) Show that the complete graph K_6 has a factorisation. [10+10 = 20]
5. (a) Find all the connected graphs with maximum degree = 2.
(b) Find all the self-complementary graphs on at most five vertices. [10+10 = 20].